Statistics

Lecture 7

Feb 19-8:47 AM

SG 12

Some Review

Suppose odds in favor of event E are 3 : 37.

1) find odds against event E.

37 : 3

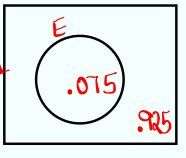
2) $P(E) = \frac{3}{3+37} = \frac{3}{40} = .075$

3) $P(\bar{E}) = \frac{37}{3+37} = \frac{37}{40} = .925$

4) Verify $P(E) + P(\bar{E}) = 1$

$.075 + .925 = 1$ ✓

5) Construct Venn Diagram



Oct 13-5:06 PM

Suppose $P(E) = .725$

1) $P(\bar{E}) = 1 - P(E) = 1 - .725 = .275$

2) odds in favor of event E.

$P(E) : P(\bar{E})$

$.725 : .275$ → 29 : 11

.725 ÷ .275 Math 1:▶Frac Enter $\frac{29}{11}$

3) odds against event E.

11 : 29

Oct 13-5:12 PM

Multiplication Rule

Keyword AND

Multiple Action event

Case I: Independent events

one outcome does not change the prob. of next outcome.

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

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ex: Flip a fair coin twice, find prob. that it lands on tails.

$P(H)=\frac{1}{2}$, $P(T)=\frac{1}{2}$

$P(\text{both are tails})=\frac{1}{2}\cdot\frac{1}{2}=\boxed{\frac{1}{4}}$

ex: Roll a fair die twice, find prob. of getting two 5's.

$P(1)=\frac{1}{6}$, $P(2)=\frac{1}{6}$, ..., $P(5)=\frac{1}{6}$ } every roll

$P(\underbrace{\text{two 5's}}_{\text{Rare event}})=\frac{1}{6}\cdot\frac{1}{6}=\boxed{\frac{1}{36}}$

$=\boxed{.028}<.05$

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A standard deck of playing cards has 52 cards, 26 red, 12 faces, and 4 aces.

Draw 2 cards with replacement

1) $P(\underbrace{\text{Two aces}}_{\text{Rare event}})=\frac{4}{52}\cdot\frac{4}{52}=\boxed{\frac{1}{169}}\approx.006<.05$

2) $P(\underbrace{\text{Two faces}}_{\text{not rare}})=\frac{12}{52}\cdot\frac{12}{52}=\boxed{\frac{9}{169}}\approx.053$

If we draw 3 cards,

$P(\text{all red})=\frac{26}{52}\cdot\frac{26}{52}\cdot\frac{26}{52}=\boxed{\frac{1}{8}}=\boxed{.125}$

Oct 13-5:26 PM

Suppose $P(A)=.5$, $P(B)=.6$, A & B are independent events

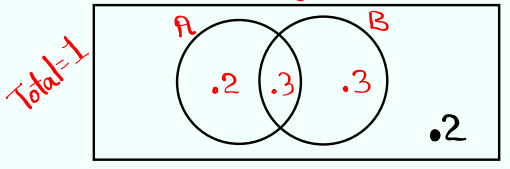
1) $P(\overline{A}) = 1 - P(A) = .5$

2) $P(\overline{B}) = 1 - P(B) = .4$

3) $P(A \text{ and } B) = P(A) \cdot P(B) = (.5)(.6) = .3$

4) $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = .5 + .6 - .3 = .8$ ✓

5) Construct Venn Diagram



6) $P(\overline{A} \text{ or } \overline{B}) = P(\overline{A \text{ and } B}) = 1 - .3 = .7$

De Morgan's Law

7) $P(\overline{A} \text{ and } \overline{B}) = P(\overline{A \text{ or } B}) = 1 - .8 = .2$

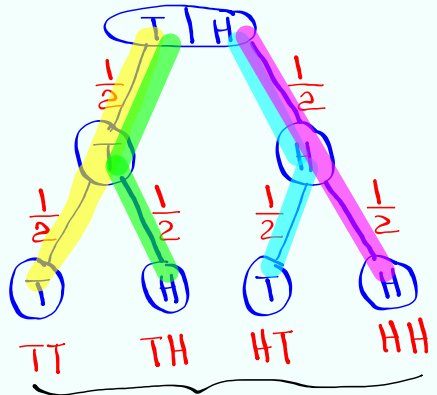
Oct 13-5:34 PM

Multiplication Rule with Tree Diagram

Flip a fair coin twice

$P(2 \text{ tails}) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

$P(2 \text{ heads}) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

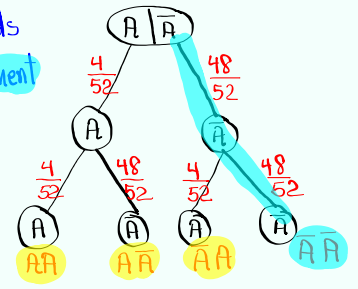


Complete list of all outcomes

Sample Space

Oct 13-5:48 PM

Standard deck of playing cards
52 cards, 4 aces.
Draw 2 cards
with replacement



P(at least 1 ace) = 1 − P(No Aces)

Total Prob.

$= 1 - \frac{48}{52} \cdot \frac{48}{52} = \frac{25}{169} \approx .148$

1 − 48 ÷ 52 × 48 ÷ 52 Math 1:▶Frac Enter

SG 12 ✓

Oct 13-5:53 PM

Multiplication Rule

SG 13

Case II: Dependent Events

One outcome changes the prob. of next outcome.

$$P(A \text{ and } B) = P(A) \cdot P(B|A)$$

(B|A: Given)

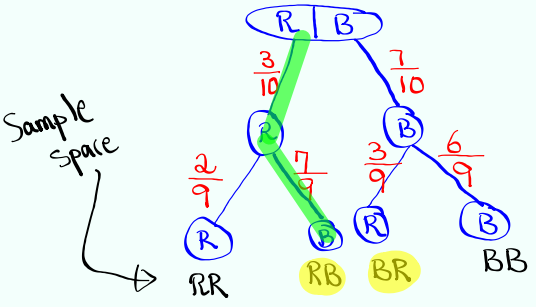
Oct 13-6:09 PM

A box has 3 Red and 7 Blue balls.

Select 2 balls, No replacement

$P(2 \text{ Red balls}) = \frac{3}{10} \cdot \frac{2}{9} = \frac{1}{15} = .067$

$P(2 \text{ Blue balls}) = \frac{7}{10} \cdot \frac{6}{9} = \frac{7}{15} = .467$



$P(1R\ 1B) = 2 \cdot \frac{3}{10} \cdot \frac{7}{9} = \frac{7}{15}$

Oct 13-6:12 PM

A box has 3 Red and 7 Blue balls.

Select 3 balls, No replacement

$P(\text{All Red}) = \frac{3}{10} \cdot \frac{2}{9} \cdot \frac{1}{8} = \frac{1}{120}$

$P(\text{All Blue}) = \frac{7}{10} \cdot \frac{6}{9} \cdot \frac{5}{8} = \frac{7}{24}$ ✓

$P(\text{all Same Color}) = \frac{1}{120} + \frac{7}{24} = \frac{3}{10}$

RRR OR BBB

RRR

Some R & Some B

BBB

P(at least 1 Red)

$= 1 - P(\text{All blue})$

$= 1 - \frac{7}{24} = \frac{17}{24}$

Oct 13-6:12 PM

$$P(A \text{ and } B) = P(A) \cdot P(B|A)$$

Given

Divide both sides by $P(A)$

$$\frac{P(A \text{ and } B)}{P(A)} = P(B|A)$$

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

Conditional Prob.

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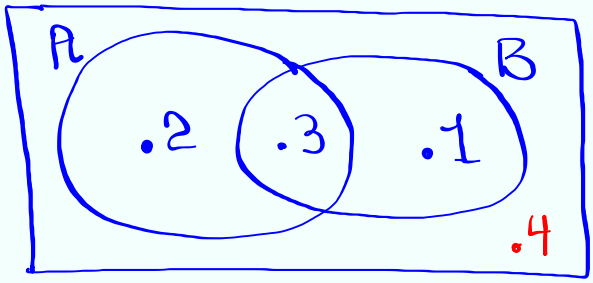
$P(A) = .5$

$P(B) = .4$

$P(A \text{ and } B) = .3$

Make Venn Diagram



Total = 1

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{.3}{.5} = \frac{3}{5} = .6$$

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{.3}{.4} = \frac{3}{4} = .75$$

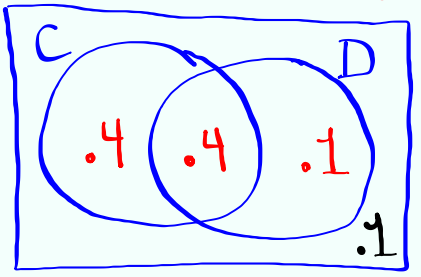
Oct 13-6:33 PM

$P(\text{Coffee}) = .8$

$P(\text{Donut}) = .5$

$P(\text{Coffee and Donut}) = .4$

Construct Venn Diag.



Total = 1

$$P(\text{Donut} \mid \text{Coffee}) = \frac{P(C \text{ and } D)}{P(C)} = \frac{.4}{.8} = \boxed{.5}$$

$$P(\text{Coffee} \mid \text{Donut}) = \frac{P(C \text{ and } D)}{P(D)} = \frac{.4}{.5} = \boxed{.8}$$

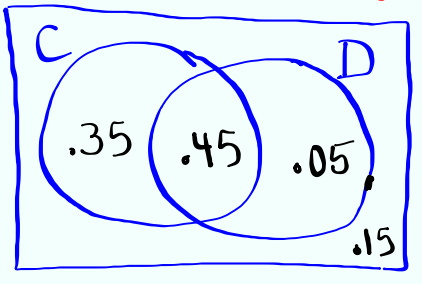
Oct 13-6:39 PM

$P(\text{Coffee}) = .8$

$P(\text{Donut}) = .5$

$P(\text{Coffee and Donut}) = .45$

Construct Venn Diag.



Total = 1

$$P(\text{Donut} \mid \text{Coffee}) = \frac{P(C \text{ and } D)}{P(C)} = \frac{.45}{.8} = .5625 \approx \boxed{.563}$$

$$P(\text{Coffee} \mid \text{Donut}) = \frac{P(C \text{ and } D)}{P(D)} = \frac{.45}{.5} = \boxed{.9}$$

Oct 13-6:39 PM

$P(A) = .5$

$P(B) = .8$

$P(A \mid B) = .6$

$P(A \mid B) = \dfrac{P(A \text{ and } B)}{P(B)}$

$.6 = \dfrac{P(A \text{ and } B)}{.8}$

Find $P(A \text{ and } B)$

Cross-Mutiply

$P(A \text{ and } B) = (.6)(.8) = .48$

$P(B|A) = \dfrac{P(A \text{ and } B)}{P(A)} = \dfrac{.48}{.5} = .96$

SG 13 ✓

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Exam 1:

1) opens Friday at 12:00 PM
   closes Saturday at 6:00 PM.

2) You have 3 hours to take it.

3) 6 pages

4) Your work must be similar to my notes.

5) Submit as you do study guides.

6) on Sunday, Read announcement to pick a date & time for Exam 1 Q & A.

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